INFLATION VIA MODIFIED GRAVITY & INFLATION IN THE MODIFIED STAROBINSKY MODEL AND IN THE EINSTEIN FRAME

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Abstract— In the present paper we consider the Starobinsky model of inflation, and find that it is connected to matter scalar field models with a non-minimal coupling to gravity. We then consider quantum induced marginal deformations of the Starobinsky action, and find that such deformations significantly shift the predicted tensor-to-scalar towards higher values. At last we discuss sources for these corrections.
In this paper, we review inflation in modified gravity, particularly F(R) gravity, based on Ref. [33]. The deviation of F(R) gravity from general relativity may be interpreted as a kind of quantum corrections in the early universe, or such a modification of gravity could be motivated by the so-called ultraviolet (UV) completion of quantum gravity. In fact, the Starobinsky inflation [32] can be regarded as inflation induced by the modification term of R2 from general relativity. We here attempt to examine inflation by the other forms of modification of gravity.

Index Terms Inflation; Starobinsky model; scalar field; Friedmann equations; Einstein-Hilbert action

1 Introduction

The past century has marked as the golden age of Cosmology. Ground-breaking observations have made Cosmology a scientific area and not a philosophical study as it used to be. Einstein's theory has the remarkable property of being able to be applied on large (cosmological) scales. It is worth noting that GR could successfully explain a great part of observations. However, General relativity alone could not fully explain all the observational data and there were many questions that remained unanswered. Thus, one can imagine that GR might not be the end of the story. For instance, the late-time accelerated expansion cannot be predicted by Einstein's theory. In particular, as can be seen by the Friedmann equations, the expansion is always decelerated for conventional matter. Therefore, the modification of Einstein's theory comes into the game. In addition, more fundamental theories such as String Theory, predict higher order terms contributing to the Gravity action when the curvature is high. Thus, both observations and theory seem to lead to the conclusion that Gravity should be modified somehow.

The standard cosmological model covers a wide class of phenomena and fits the current observational tests with great success. However, this model has problems of 1,2 the initial singularity, horizon, flatness and monopoles in the early period of the universe. These problems can be solved if we assume that the primordial universe starts with a very fast expansion, denominated inflation by Guth in 1981.3

An essential natural inflationary scenario is one in which inflation is driven by quantum corrections to the Einstein-Hilbert action, suggested by Starobinsky in 1980.4 The Starobinsky model is based on the semi classical approach to quantum field theory (QFT) in curved space-time. Within this theory the metric is treated as a classical background for the quantum dynamics of the matter fields. This approach presents a consistent theory at energies of a few orders of magnitude below the Planck scale.5,6

In the original Starobinsky model, inflation is a consequence of the quantum effects of massless matter fields.4 The model assumes a non-minimal conformal coupling between the scalar field and gravity, $\xi = 1/6$. In this case, the mass-less matter fields are conformally invariant having a traceless stress tensor at the classical level. However, the one-loop contributions create a trace anomaly which changes the dynamics of the conformal factor of the metric (see Refs. 4, 7, 8) and also the metric and density perturbations.9,10,11 An alternative option is to apply the effective action method, using the conformal anomaly to calculate the induced effective action.6,12–15 Inflation naturally arises from the total action which is obtained from the sum of the anomaly-induced effective action to the classical terms, including the Einstein-Hilbert one.16–18

An alternative version of the Starobinsky model was proposed in Refs. 19, 20,21. The main advantage of this modified version is that inflation starts in the stable regime which is after words interpolating to an unstable regime at the end of inflation.4,8,17 The modified Starobinsky (MSt) model is a natural extension of the Starobinsky model. In the MSt version, inflation is due to the contribution of the quantum effects of both massless conformal and massive matter fields.19–21 The massive theory is not conformally invariant at the classical level due to the masses of the scalar and fermion fields. However, using a conformal description, the massive matter fields become conformally invariant and we can use the conformal anomaly method to derive the effective action.

The Starobinsky model is a natural inflationary scenario in which inflation arises due to quantum effects of the massless matter fields. A modified version of the Starobinsky model takes the masses of matter fields and the cosmological constant, Λ, into account. The equations of motion become much more complicated however approximate analytic and numeric solutions are possible. In the MSt model, inflation starts due to the super symmetric (SUSY) particle content of the underlying theory and the transition to the radiation dominated epoch occurs due to the relatively heavy s-particles decoupling. For $\Lambda = 0$ the inflationary solution is stable until the last stage, just before decoupling

INFLATION VIA MODIFIED GRAVITY

This requires one to go beyond standard Einstein gravity and consider modified versions, for example in the context of f(R)-theories.

In these theories the action is,

$$S = \int d^4x\sqrt{-g}\,\frac{M_P^2}{2}\,f(R) + \int d^4x\, L_M(g_{\mu\upsilon}, \psi_M) \quad \ldots\ldots\ldots \quad (1)$$

Where f (R) is an arbitrary function of the Ricci scalar R and LM is a matter Lagrangian which is minimally coupled to gravity. This includes the Starobinsky model of inflation, which is one of the earliest models of inflation. The Starobinsky model features an R2-term added to the Einstein-Hilbert action, $f(R) = R + \frac{R^2}{6M^2}$ … … … (2)

Where M is a new mass scale. We consider the Starobinsky model of Inflation in detail below. We begin our discussion by considering the field equations associated to the general action. These may be found by varying the action with respect to $g_{\mu\upsilon}$,

$$F(R)R_{\mu\vartheta} - \frac{1}{2}f(R)g_{\mu\vartheta} - \nabla_\mu\nabla_\vartheta F(R) + g_{\mu\vartheta}\Box F(R) = M_P^{-2}T_{\mu\vartheta}^M \quad \ldots\ldots\ldots \ (3)$$

Where $F(R) \equiv \frac{\delta f}{\delta R}$ & $T_{\mu\vartheta}^M$ is the energy-momentum tensor of the matter fields. We obtain the Starobinsky-Einstein equation $G_{\mu\vartheta} \equiv R_{\mu\vartheta} - \frac{1}{2}Rg_{\mu\vartheta} = -8\pi G T_{\mu\vartheta}$ by setting f(R) =R and F(R) = 1.

Now from equation (3) we have,

$$F(R)R_{\mu\vartheta} - \frac{1}{2}f(R)g_{\mu\vartheta} - \nabla_\mu\nabla_\vartheta F(R) + g_{\mu\vartheta}\Box F(R) = M_P^{-2}T_{\mu\vartheta}^M$$

$$\Rightarrow F(R)g^{\mu\vartheta}R_{\mu\vartheta} - \frac{1}{2}f(R)g^{\mu\vartheta}g_{\mu\vartheta} - \nabla_\mu\nabla_\vartheta F(R)g^{\mu\vartheta} + g_{\mu\vartheta}g^{\mu\vartheta}\Box F(R) = M_P^{-2}g^{\mu\vartheta}T_{\mu\vartheta}^M \quad \ldots\ldots..(4)$$

Here,

$$F(R)g^{\mu\vartheta}R_{\mu\vartheta} = F(R)[g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33}]$$

$$= F(R)\left[(-1)(-\frac{3\ddot{R}}{R}) + \frac{1}{R^2}(2\dot{R}^2 + R\ddot{R}) + \frac{1}{R^2r^2}r^2(2\dot{R}^2 + R\ddot{R}) + \frac{1}{R^2r^2\sin^2\theta}r^2\sin^2\theta(2\dot{R}^2 + R\ddot{R})\right]$$

$$= F(R)\left(\frac{3\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{\ddot{R}}{R} + \frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{2\dot{R}^2}{R^2} + \frac{\ddot{R}}{R}\right)$$

$$= F(R)\left[6(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2})\right] \quad = F(R)R$$

$$\frac{1}{2}f(R)g^{\mu\vartheta}g_{\mu\vartheta} = \frac{1}{2}f(R)[g^{00}g_{00} + g^{11}g_{11} + g^{22}g_{22} + g^{33}g_{33}]$$

$$= \frac{1}{2}f(R)\left[(-1)(-1) + \frac{1}{R^2}R^2 + \frac{1}{R^2r^2}R^2r^2 + \frac{1}{R^2r^2\sin^2\theta}R^2r^2\sin^2\theta\right]$$

$$= \frac{1}{2}f(R)(1+1+1+1) \quad = \frac{4}{2}f(R)$$

$$g^{\mu\vartheta}\nabla_\mu\nabla_\vartheta F(R) = g^{00}\nabla_0\nabla_0 F + g^{11}\nabla_1\nabla_1 F + g^{22}\nabla_2\nabla_2 F + g^{33}\nabla_3\nabla_3 F$$

$$= g^{00}(\delta_0\delta_0 F - \Gamma_{00}^0\dot{F}) + g^{11}(\delta_1\delta_1 F - \Gamma_{11}^0\dot{F}) + g^{22}(\delta_2\delta_2 F - \Gamma_{22}^0\dot{F}) + g^{33}(\delta_3\delta_3 F - \Gamma_{33}^0\dot{F})$$

$$= (-1)\left(\frac{d^2F}{dt^2} - 0\right) + \frac{1}{R^2}\left(\frac{d^2F}{dr^2} - R\dot{R}\frac{dF}{dt}\right) + \frac{1}{R^2r^2}\left(\frac{d^2F}{d\theta^2} - R\dot{R}r^2\frac{dF}{dt}\right) + \frac{1}{R^2r^2\sin^2\theta}\left(\frac{d^2F}{d\varphi^2} - R\dot{R}r^2\sin^2\theta\frac{dF}{dt}\right)$$

$$= -\frac{d^2F}{dt^2} + \frac{1}{R^2}\left(0 - R\dot{R}\frac{dF}{dt}\right) + \frac{1}{R^2r^2}\left(0 - R\dot{R}r^2\frac{dF}{dt}\right) + \frac{1}{R^2r^2\sin^2\theta}\left(0 - R\dot{R}r^2\sin^2\theta\frac{dF}{dt}\right)$$

$$= -\frac{d^2F}{dt^2} - \frac{\dot{R}}{R}\frac{dF}{dt} - \frac{\dot{R}}{R}\frac{dF}{dt} - \frac{\dot{R}}{R}\frac{dF}{dt} = -\frac{d^2F}{dt^2} - 3H\frac{dF}{dt}$$

$$= -(\frac{d^2}{dt^2} + 3H\frac{d}{dt})F \quad = \Box F \quad [\text{for } \Box = -(\frac{d^2}{dt^2} + 3H\frac{d}{dt})F\,]$$

And,

$$g_{\mu\vartheta}g^{\mu\vartheta}\Box F(R) = \Box F(R)[g^{00}g_{00} + g^{11}g_{11} + g^{22}g_{22} + g^{33}g_{33}]$$

$$= \Box F(R)(1+1+1+1)$$

$$= 4\Box F(R)$$

Now from equation (4) we get,

$$F(R)R - \frac{4}{2}f(R) - \Box F(R) + 4\Box F(R) = M_P^{-2}g^{\mu\vartheta}T_{\mu\vartheta}^M$$

$$\Rightarrow F(R)R - 2f(R) + 3\Box F(R) = M_P^{-2}g^{\mu\vartheta}T_{\mu\vartheta}^M \quad \ldots\ldots\ldots \ (5)$$

This reveals an extra propagating scalar degree of freedom $\psi = F(R)$ as compared to standard Einstein gravity. We will soon see that this extra scalar degree of freedom may be used to drive inflation. In Einstein gravity the term F(R) vanishes and $R = -M_P^{-2}g^{\mu\vartheta}T_{\mu\vartheta}^M$ such that the Ricci scalar is determined by the matter content in the standard manner.

In the following we consider vacuum solutions with $T_{\mu\vartheta}^M = 0$.

In equation (3) we will consider the effects of integrating out matter fields. Also we consider flat FRW space-time (K=0) with metric,

$$ds^2 = -dt^2 + R^2(t)\delta_{ij}dx^i dx^j \quad \ldots\ldots\ldots \ (6)$$

We have,

$$x^i, x^j = (x^0, x^1, x^2, x^3) = (t, r, \theta, \varphi) \quad \& \quad g_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

From the metric tensor,

$$ds^2 = g_{ij}dx^i dx^j$$

$$\Rightarrow ds^2 = g_{00}dx^0dx^0 + g_{11}dx^1dx^1 + g_{22}dx^2dx^2 + g_{33}dx^3dx^3$$

$$\Rightarrow ds^2 = g_{00}(dx^0)^2 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2$$

$$\Rightarrow ds^2 = -dt^2 + R^2dr^2 + R^2r^2d\theta^2 + R^2r^2\sin^2\theta d\varphi^2$$

$$\therefore ds^2 = -dt^2 + R^2[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)] \quad \& \quad R = \frac{6(R\ddot{R} + \dot{R}^2)}{R^2}$$

We know,

$$H = \frac{\dot{R}}{R}$$

$$\Rightarrow \frac{d}{dt}(H) = \frac{d}{dt}(\frac{\dot{R}}{R})$$

$$\Rightarrow \dot{H} = \frac{R\ddot{R} - \dot{R}^2}{R^2} = \frac{R\ddot{R}}{R^2} - \frac{\dot{R}^2}{R^2} = \frac{\ddot{R}}{R} - H^2$$

$$\therefore \frac{\ddot{R}}{R} = \dot{H} + H^2$$

$$\therefore R = \frac{6(R\ddot{R} + \dot{R}^2)}{R^2} = 6(\frac{R\ddot{R}}{R^2} + \frac{\dot{R}^2}{R^2}) = 6(\frac{\ddot{R}}{R} + H^2) = 6(\dot{H} + H^2 + H^2) = 6(\dot{H} + 2H^2)$$

The Ricci scalar is,

$$R = (\dot{H} + 2H^2) \quad \dots\dots\dots (7)$$

With H the Hubble constant .Since we are studying inflation we are interested in (quasi) de Sitter solutions with H and R constant. In this case the term F(R) vanished from the trace equation which then reads,

$$0 + F(R)R - 2f(R) = 0$$

$$\Rightarrow F(R)R - 2f(R) = 0 \quad \dots\dots\dots (8)$$

$$\Rightarrow F(R)R = 2f(R) = F(R)R = \frac{\delta f}{\delta R}R \qquad \left[\because F(R) = \frac{\delta f}{\delta R}R\right]$$

$$\Rightarrow 2\frac{\delta R}{R} = \frac{\delta f}{f}$$

$$\Rightarrow \ln f = 2\ln R + \ln R_0$$

$$\Rightarrow f(R) = R_0R^2$$

$$\therefore f(R) \propto R^2$$

The model $f(R) \propto R^2$ solves this condition & gives rise to an exact de-sitter solution.
We may consider this is as a correction to Einstein gravity & write,

$$f(R) = R + \frac{R^2}{6M^2}$$

$$\Rightarrow \frac{R}{2}F(R) = R(1 + \frac{R}{6M^2})$$

$$\therefore F(R) = 1 + \frac{R}{3M^2} \qquad \dots\dots\dots (9)$$

Where M is a mass scale .Then at high R-values where the R^2-term dominates we obtain quasi de Sitter expansion $F(R)R - 2f(R) \approx 0$. This is the famous Starobinsky model of inflation. During inflation R decreases such that Inflation ends when the quadratic term becomes smaller than the linear term R ~M^2.
Now we first insert the Starobinsky model and the FRW-metric in the field equations (3) then we can get the following calculation,

$$R_{\mu\vartheta} - \frac{1}{2}Rg_{\mu\vartheta} = -8\pi GT_{\mu\vartheta}$$

For μ = υ = 0,

$$R_{00} - \frac{1}{2}Rg_{00} = -8\pi GT_{00}$$

$$\Rightarrow \frac{-3\ddot{R}}{R} - \frac{1}{2}(-1)\frac{6(R\ddot{R} + \dot{R}^2)}{R^2} = -8\pi GT_{00}$$

$$\Rightarrow \frac{-3\ddot{R}}{R} + \frac{3R\ddot{R}}{R^2} + \frac{3\dot{R}^2}{R^2} = -8\pi GT_{00}$$

$$\Rightarrow \frac{-3\ddot{R}}{R} + \frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = -8\pi GT_{00}$$

$$\Rightarrow \frac{3\dot{R}^2}{R^2} = -M_P^{-2}T_{00} \qquad \left[\because M_P^2 = \frac{1}{\sqrt{8\pi G}}\right]$$

$$\therefore M_P^{-2}T_{00} = -\frac{3\dot{R}^2}{R^2}$$

Now, $F(R)R_{\mu\vartheta} - \frac{1}{2}f(R)g_{\mu\vartheta} - \nabla_\mu\nabla_\vartheta F(R) + g_{\mu\vartheta}\,\Box F(R) = M_P^{-2}T_{\mu\vartheta}^M$

For μ = υ = 0,

$$F(R)R_{00} - \frac{1}{2}f(R)g_{00} - \nabla_0\nabla_0F(R) + g_{00}\,\Box\,F(R) = M_P^{-2}T_{00}^M$$

$$\Rightarrow F(R)(-\frac{3\ddot{R}}{R}) - \frac{R}{4}F(R)(-1) - \delta_0\delta_0F + \Gamma_{00}^0\dot{F} + (-1)\{-(\frac{d^2}{dt^2} + 3H\frac{d}{dt})F\} = M_P^{-2}T_{00}^M$$

$$\Rightarrow -\frac{3\ddot{R}}{R}F(R) + \frac{R}{4}F(R) - \frac{d^2F}{dt^2} + 0 + \frac{d^2F}{dt^2} + 3H\frac{dF}{dt} = M_P^{-2}T_{00}^M$$

$$\Rightarrow (\frac{R}{4} - \frac{3\ddot{R}}{R})F(R) + 3H\frac{dF}{dt} = M_P^{-2}T_{00}^M$$

$$\Rightarrow \left[\frac{6(2H^2 + \dot{H})}{4} - 3(\dot{H} + H^2)\right]F(R) + 3H\frac{d}{dt}\left(1 + \frac{R}{3M^2}\right) = M_P^{-2}T_{00}^M$$

$$\Rightarrow (3H^2 + \frac{3}{2}\dot{H} - 3\dot{H} - 3H^2)\left(1 + \frac{R}{3M^2}\right) + 3H\frac{d}{dt}\left(1 + \frac{R}{3M^2}\right) = M_P^{-2}T_{00}^M$$

$$\Rightarrow -\frac{3}{2}\dot{H}(1 + \frac{R}{3M^2}) + 3H\frac{d}{dt}\left(1 + \frac{R}{3M^2}\right) = M_P^{-2}T_{00}^M$$

$$\Rightarrow -\frac{3}{2}\dot{H}\left(1 + \frac{6(2H^2 + \dot{H})}{3M^2}\right) + 3H\frac{d}{dt}[1 + \frac{6(2H^2 + \dot{H})}{3M^2}] = -\frac{3\dot{R}^2}{R^2}$$

$$\Rightarrow -\frac{3}{2}\dot{H} - \frac{6H^2\dot{H}}{M^2} - \frac{3\dot{H}^2}{M^2} + \frac{3H}{M^2}\frac{d}{dt}(M^2 + 4H^2 + 2\dot{H}) = -\frac{3\dot{R}^2}{R^2}$$

$$\Rightarrow -\frac{3}{2}\dot{H} - \frac{6H^2\dot{H}}{M^2} - \frac{3\dot{H}^2}{M^2} + \frac{3H}{M^2}\times 8H\dot{H} + \frac{3H}{M^2}\times 2\ddot{H} = -3H^2$$

$$\Rightarrow -\frac{3}{2}\dot{H} - \frac{6H^2\dot{H}}{M^2} - \frac{3\dot{H}^2}{M^2} + \frac{24\dot{H}H^2}{M^2} + \frac{6\ddot{H}H}{M^2} = -3H^2$$

$$\Rightarrow \left(-\frac{3}{2}\dot{H} - \frac{3\dot{H}^2}{M^2} + \frac{18\dot{H}H^2}{M^2} + \frac{6\ddot{H}H}{M^2}\right)\frac{M^2}{6H} = -3H^2 \times \frac{M^2}{6H}$$

$$\Rightarrow -\frac{\dot{H}M^2}{4H} - \frac{1}{2}\frac{\dot{H}^2}{H} + 3H\dot{H} + \ddot{H} = -\frac{1}{2}M^2H$$

$$\therefore \ddot{H} - \frac{\dot{H}^2}{2H} + \frac{1}{2}M^2H + 3H\dot{H} = \frac{\dot{H}M^2}{4H}$$

Again, for μ = υ = 1 $\qquad R_{11} - \frac{1}{2}Rg_{11} = -8\pi GT_{11}$,

$$\Rightarrow R\ddot{R} + 2\dot{R}^2 - \frac{1}{2}R^2\,\frac{6(R\ddot{R} + \dot{R}^2)}{R^2} = -8\pi GT_{11}$$

$$\Rightarrow R\ddot{R} + 2\dot{R}^2 - 3R\ddot{R} - 3\dot{R}^2 = -8\pi GT_{11}$$

$$\Rightarrow -2R\ddot{R} - \dot{R}^2 = -8\pi GT_{11}$$

$$\therefore 2R\ddot{R} + R^2 = M_P^{-2}T_{11} \qquad \left[\because M_P^2 = \frac{1}{\sqrt{8\pi G}}\right]$$

Now, for μ = υ = 1,

$$F(R)R_{11} - \frac{1}{2}f(R)g_{11} - \nabla_1\nabla_1F(R) + g_{11}\Box F(R) = M_P^{-2}T_{11}^M$$

$$\Rightarrow (2\dot{R}^2 + \ddot{R}R)F(R) - \frac{R}{4}R^2F(R) - \delta_1\delta_1F + \Gamma_{11}^0\dot{F} + R^2\{-(\frac{d^2}{dt^2} + 3H\frac{d}{dt})F\} = M_P^{-2}T_{11}^M$$

$$\Rightarrow (2\dot{R}^2 + \ddot{R}R - \frac{R^3}{4})F(R) - \frac{d^2F}{dr^2} + R\dot{R}\dot{F} - R^2\frac{d^2F}{dt^2} - 3HR^2\frac{dF}{dt} = M_P^{-2}T_{11}^M$$

$$\Rightarrow (2\dot{R}^2 + \ddot{R}R - \frac{R^3}{4})F(R) - 0 - 2R\dot{R}\frac{dF}{dt} - R^2\frac{d^2F}{dt^2} = M_P^{-2}T_{11}^M$$

$$\Rightarrow (2\dot{R}^2 + \ddot{R}R - \frac{R^3}{4})(1 + \frac{R}{3M^2}) - 2R\dot{R}\frac{d}{dt}(1 + \frac{R}{3M^2}) - R^2\frac{d^2}{dt^2}(1 + \frac{R}{3M^2}) = M_P^{-2}T_{11}^M$$

$$\Rightarrow 2\dot{R}^2 + \ddot{R}R - \frac{R^3}{4} + \frac{2\dot{R}^2R}{3M^2} + \frac{R^2\ddot{R}}{3M^2} - \frac{R^4}{12M^2} - \frac{2\dot{R}^2R}{3M^2} - \frac{R^2\ddot{R}}{3M^2} = M_P^{-2}T_{11}^M$$

$$\Rightarrow 2\dot{R}^2 + R\ddot{R} - \frac{R^3}{4} - \frac{R^4}{12M^2} = \dot{R}^2 + 2R\ddot{R}$$

$$\Rightarrow \dot{R}^2 - R\ddot{R} - \frac{R^3}{4} - \frac{R^4}{12M^2} = 0$$

$$\Rightarrow \ddot{R} - \frac{\dot{R}^2}{R} + \frac{R^2}{4} + \frac{R^3}{12M^2} = 0$$

$$\Rightarrow \ddot{R} - \frac{\dot{R}}{R}\dot{R} + \frac{R^2}{4}\left(1 + \frac{R}{3M^2}\right) = 0$$

$$\therefore \ddot{R} - H\dot{R} + \frac{R^2}{4}F = 0$$

So finally we get,

$$\left.\begin{aligned} &\ddot{H} - \frac{\dot{H}^2}{2H} + \frac{1}{2}M^2H + 3H\dot{H} = \frac{\dot{H}M^2}{4H} \\ &\ddot{R} - H\dot{R} + \frac{R^2}{4}F = 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots (10)$$

The first equation is the (0,0)-component which have been inserted in the (i, i)-component to obtain the second equation. When deriving these equations it is useful to know that the FRW-metric yields,

$$\Box F = \frac{1}{\sqrt{-g}}\delta_\mu(\sqrt{-g}\,g^{\mu\vartheta}\delta_\mu F) = -(\frac{d^2}{dt^2} + 3H\frac{d}{dt})F$$

$$\nabla_\mu\nabla_\vartheta F = \delta_\mu\delta_\vartheta F - \Gamma_{\mu\vartheta}^0\dot{F} \ , \ \Gamma_{00}^0 = 0 \ \ \& \ \ \Gamma_{ij}^0 = R\dot{R}\delta_{ij}$$

As we did earlier, we quantify slow-roll by smallness of the Hubble slow-roll parameters,

$$\varepsilon_H = \left|\frac{\dot{H}}{H^2}\right| << 1 \quad , \quad \eta_H = \left|\frac{\ddot{H}}{H\dot{H}}\right| << 1$$

The first two terms in equation (10) may then be neglected. From equation (9) we find that $R \approx 12H^2$, hence $\ddot{R}$ can also be neglected. The slow-roll approximation then becomes,

$$0 + 0 + \frac{1}{2}M^2H = -3H\dot{H} + 0$$

$$\therefore \dot{H} \approx -\frac{1}{6}M^2$$

The term may readily be integrated to obtain the slow-roll solution,

$$\frac{dH}{dt} \approx -\frac{1}{6}M^2$$

$$\Rightarrow dH \approx -\frac{1}{6}M^2dt$$

Integrating,

$$H \approx H_i - \frac{1}{6}M^2\int_{t_i}^{t}dt$$

$$\therefore H \approx H_i - \frac{1}{6}M^2(t - t_i)$$

$$R \approx R_i\exp[H_i(t - t_i) - \frac{1}{12}M^2(t - t_i)^2]$$

$$R \approx 12H^2 - M^2 \qquad \ldots\ldots\ldots (11)$$

Where i denotes the initial conditions .It can be shown that the slow-roll trajectory is an attractor in phase space and hence the further evolution is largely independent on the Initial conditions, as we discussed in section before. Accelerated expansion occurs as long as the slow-roll parameter εH is smaller than unity.

$$\varepsilon_H = -\frac{\dot{H}}{H^2} \cong \frac{M^2}{6H^2} \qquad \dots\dots\dots (12)$$

Hence inflation occurs for $H^2 > M^2$. Inflation ends when εH =1 i.e. $H_{end} \cong \frac{M}{\sqrt{6}}$. It follows that this corresponds to the time at which the Ricci scalar decreases to $R \sim M^2$.

II.STAROBINSKY INFLATION IN THE EINSTEIN FRAME

The f(R) theory,

$$F(R)R - 2f(R) + 3\Box F(R) = M_P^{-2} g^{\mu\vartheta} T^M_{\mu\vartheta} \qquad \dots\dots\dots (13)$$

This equation may be cast in a form that features a potential for the extra scalar degree of freedom which appeared above. This can be done by considering the following linear representation in terms of a new field y.

$$S = \int d^4x \sqrt{-g}\, \frac{M_P^2}{2} [f(y) + f'(y)(R-y)] \qquad \dots\dots\dots (14)$$

We set $T^M_{\mu\vartheta} = 0$ since we will insert the Starobinsky model shortly .The equation of motion for y is,

$$f''(y)(R-y) = 0$$

If $f''(y) \neq 0$ it follows that y = R & we recover the original action equation (13).

By inserting the scalar degree of freedom,

$$\Psi \equiv f'(y) = F(y)$$

$$\Rightarrow f'(y) = \Psi \qquad \dots\dots\dots (15)$$

$$\Rightarrow f(y) = \Psi y \qquad \text{[Integrating w.r.to y]}$$

$$\Rightarrow f(R) = \Psi R \qquad \text{[for y = R]}$$

$$\Rightarrow f(y) = \Psi R \qquad \dots\dots\dots (16)$$

Again,

$$f'(y)(R-y) = (R-y)\Psi = R\Psi - y\Psi = f(y) - y\Psi$$

$$\Rightarrow f'(y)(R-y) = f(y(\Psi)) - y(\Psi)\Psi$$

$$\Rightarrow f'(y)(R-y) = -[y(\Psi)\Psi - f(y(\Psi))]$$

Putting these values in equation (14) we get,

$$S = \int d^4x \sqrt{-g}\, \frac{M_P^2}{2} [\Psi R - \{y(\Psi)\Psi - f(y(\Psi))\}]$$

$$\Rightarrow S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 \Psi R - \frac{1}{2} M_P^2 \{y(\Psi)\Psi - f(y(\Psi))\} \right]$$

$$\Rightarrow S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 \Psi R - V(\Psi) \right] \qquad \dots\dots\dots (17)$$

Where,

$$V(\Psi) = \frac{1}{2} M_P^2 [y(\Psi)\Psi - f(y(\Psi))]$$

Hence we have obtained and action for the scalar degree of freedom ψ with potential V (ψ) which is equivalent to the f (R)-theory. It appears to have the same form as the non-minimally coupled models we considered earlier

$$S_j = \int d^4x \sqrt{-g} \left[\frac{M_P^2 + \xi\varphi^2}{2} R - \frac{1}{2} g^{\mu\vartheta} \delta_\mu \varphi \delta_\vartheta \varphi - V(\varphi) \right]$$

except that there is no kinetic term. We will discuss similarities and differences within the framework of Starobinsky inflation shortly. First we proceed by performing a conformal transformation. To do this it is convenient to reinsert F(R) and write the action in the form,

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} FR - V \right] \qquad \dots\dots\dots (17)$$

Let us briefly repeat the steps of the conformal transformation.

The metric and Ricci scalar transform as

$$g_{\mu\vartheta} \rightarrow \hat{g}_{\mu\vartheta} = \Omega^2 g_{\mu\vartheta}$$

$$R = \Omega^2 [\hat{R} + 6\Box \ln\Omega - 6\hat{g}^{\mu\vartheta} (\delta_\mu \ln\Omega)(\delta_\vartheta \ln\Omega)] \qquad \dots\dots\dots (19)$$

The transformed action then reads,

$$S = \int d^4x \sqrt{-\hat{g}} [\frac{M_P^2}{2} F\Omega^2 [\hat{R} + 6\Box \ln\Omega - 6\hat{g}^{\mu\vartheta} (\delta_\mu \ln\Omega)(\delta_\vartheta \ln\Omega) - V]$$

$$\Rightarrow S = \int d^4x \sqrt{-\hat{g}} [\frac{M_P^2}{2} F\Omega^{-2} [\hat{R} + 6\Box \ln\Omega - 6\hat{g}^{\mu\vartheta} (\delta_\mu \ln\Omega)(\delta_\vartheta \ln\Omega) - \Omega^{-4} V] \qquad \dots\dots\dots (20)$$

We land in the Einstein frame where the action is linear in $\hat{R}$ if we choose $\Omega^2 = F$

We also see that the action may be canonically normalized by the field redefinition,

$$\chi = \sqrt{\frac{3}{2}} M_P \ln F$$

Defining the Einstein frame potential $U(\chi)$ as,

$$U(\chi) = \Omega^{-4} V = \frac{V}{F^2} = \frac{1}{F^2} \cdot \frac{1}{2} M_P^2 [y(\psi)\psi - f(y(\psi))] = \frac{1}{2F^2} M_P^2 (y\psi - f)$$

$$\therefore U(\chi) = \frac{1}{2F^2} M_P^2 (RF - f) \quad [\because \Psi(y) = F(y) \;\&\; y = R] \qquad \dots\dots\dots (21)$$

The action finally takes the form from (20) we get,

$$S_E = \int d^4x \sqrt{-\hat{g}} [\frac{M_P^2}{2} F\Omega^{-2} R - \frac{M_P^2}{2} F\Omega^{-2} \{6\Box \ln\Omega - 6\hat{g}^{\mu\vartheta} (\delta_\mu \ln\Omega)(\delta_\vartheta \ln\Omega)\} - \Omega^{-4} V]$$

$$= \int d^4x \sqrt{-\hat{g}} [\frac{M_P^2}{2} \Omega^2 \Omega^{-2} R - \frac{M_P^2}{2} \Omega^2 \Omega^{-2} \{6\Box \ln\Omega - 6\hat{g}^{\mu\vartheta} (\delta_\mu \ln\Omega)(\delta_\vartheta \ln\Omega)\} - \Omega^{-4} V]$$

$$\therefore S_E = \int d^4x \sqrt{-\hat{g}} [\frac{M_P^2}{2} R - \frac{1}{2} \hat{g}^{\mu\vartheta} \delta_\mu \chi \delta_\vartheta \chi - U(\chi)] \qquad \dots\dots\dots (22)$$

We may now follow the same steps as earlier, and analyze inflation using the Einstein frame potential within the standard slow-roll paradigm. We proceed by inserting the Starobinsky model,

$$f(R) = R + \frac{R^2}{6M^2} \qquad \Rightarrow F(R) = 1 + \frac{R}{3M^2} \qquad \ldots\ldots\ldots (23)$$

The field redefinition then reads

$$\chi = \sqrt{\frac{3}{2}} M_P \ln(1 + \frac{R}{3M^2}) \qquad \text{[Using (23)]}$$

Using this relation, the Einstein frame potential i.e. equation (21) becomes

$$U(\chi) = \frac{M_P^2 (FR - f)}{2F^2} = \frac{M_P^2}{2}\left(\frac{R(1+\frac{R}{3M^2}) - (R + \frac{R^2}{6M^2})}{(1+\frac{R}{3M^2})^2} \right)$$

$$\Rightarrow U(\chi) = \frac{M_P^2}{2}\left(\frac{R + \frac{R^2}{3M^2} - R - \frac{R^2}{6M^2}}{(1+\frac{R}{3M^2})^2} \right) = \frac{M_P^2}{2} \times \frac{R^2}{6M^2} \times \frac{1}{(1+\frac{R}{3M^2})^2}$$

$$\Rightarrow U(\chi) = \frac{M_P^2}{2} \times \frac{1}{6M^2} \times 9M^4 (F-1)^2 \frac{1}{F^2} \qquad \text{[by using (23)]}$$

$$\Rightarrow U(\chi) = \frac{3}{4} M_P^2 M^2 (1 - \frac{1}{F})^2$$

$$\Rightarrow U(\chi) = \frac{3}{4} M_P^2 M^2 \left(1 - \frac{1}{1 + \frac{R}{3M^2}} \right)^2$$

$$\Rightarrow U(\chi) = \frac{3}{4} M_P^2 M^2 \left[1 - \left(1 + \frac{R}{3M^2} \right)^{-1} \right]^2$$

$$\Rightarrow U(\chi) = \frac{3}{4} M_P^2 M^2 \left[1 - \exp\left\{ \ln\left(1 + \frac{R}{3M^2} \right)^{-1} \right\} \right]^2$$

$$\Rightarrow U(\chi) = \frac{3}{4} M_P^2 M^2 \left[1 - \exp\left\{ -\ln\left(1 + \frac{R}{3M^2} \right) \right\} \right]^2$$

$$\Rightarrow U(\chi) = \frac{3}{4} M_P^2 M^2 \left[1 - \exp\left\{ \frac{-2}{\sqrt{6} M_P} \times \sqrt{\frac{2}{3}} M_P \ln\left(1 + \frac{R}{3M^2} \right) \right\} \right]^2$$

$$\therefore U(\chi) = \frac{3}{4} M_P^2 M^2 \left(1 - \exp\left[\frac{-2\chi}{\sqrt{6} M_P} \right] \right)^2$$

Except for the overall coefficient, this is the same as the large field limit of the quartic potential with non-minimal coupling. The two potentials coincide if we make the identification

$$U(\chi(\varphi)) = \Omega\text{-}4 \;\; V(\varphi) \cong \frac{\lambda M_P^4}{4\xi^2} \left(1 - \frac{M_P^2}{\xi \varphi^2} \right)^2$$

$$U(\chi) \cong \frac{\lambda M_P^4}{4\xi^2} \left(1 - \exp\left[\frac{-2\chi}{\sqrt{6} M_P} \right] \right)^2$$

The two potentials coincide if we make the identification,

$$M^2 = \frac{\lambda}{3\xi^2} M_P^2$$

III.Inflation In the modified Starobinsky Model

We encode these ideas as deformations of the Starobinsky action

$$S = \int d^4x \sqrt{-g} [\frac{M_P^2}{2} R + a M_P^{4\alpha} R^{2(1-\alpha)}] \qquad \ldots\ldots\ldots (24)$$

Where a is now a dimensionless parameter. Now replacing a with the dimension full parameter

$$a \to \frac{1}{12 M^2 M_P^2} \qquad \& \qquad \alpha = 0$$

Then we get,

$$S = \int d^4x \sqrt{-g} [\frac{M_P^2}{2} R + \frac{1}{12 M_P^2 M^2} M_P^{4\times 0} R^{2(1-0)}]$$

$$\Rightarrow S = \int d^4x \sqrt{-g} [\frac{M_P^2}{2} R + \frac{1}{12 M_P^2 M^2} M_P^0 R^2]$$

$$\Rightarrow S = \int d^4x \sqrt{-g} \frac{M_P^2}{2} [R + \frac{R^2}{6 M_P^4 M^2}]$$

The equivalence between the Starobinsky model and non-minimally coupled large field φ4 – inflation allows us to map the deformed Starobinsky action into the model with potential $\lambda(\frac{\varphi}{\Lambda})^{4\gamma}$ which we considered,

$$S_j = \int d^4x \sqrt{-g} \left[\frac{M_P^2 + \xi\varphi^2}{2} R - \frac{1}{2} g^{\mu\vartheta} \delta_\mu \varphi \delta_\vartheta \varphi - \lambda \varphi^4 (\frac{\varphi}{\Lambda})^{4\gamma} \right] \qquad \ldots\ldots\ldots (25)$$

During Inflation the kinetic term (i.e. $g^{\mu\vartheta} \delta_\mu \varphi \delta_\vartheta \varphi$) is negligible, which as we have seen, corresponds to the large field regime $\varphi >> \frac{M_P}{\sqrt{\xi}}$ with large non-minimal coupling ξ. The action then reads,

$$S_J = \int d^4x \sqrt{-g} \left[\frac{M_P^2 + \xi\varphi^2}{2} R - \lambda \varphi^4 (\frac{\varphi}{\Lambda})^{4\gamma} \right] \qquad \ldots\ldots\ldots (26)$$

This is equivalent to the linear representation of the deformed Starobinsky action (25) if we make the following identifications,

This is equivalent to the linear representation of the deformed Starobinsky action (25) if we make the following identifications,

$$\alpha = \frac{\gamma}{1+2\gamma}, \qquad a^{1+2\gamma} = \left(\frac{\xi}{4} \frac{(1+2\gamma)}{(1+\gamma)} \right)^{2(1+\gamma)} \frac{1}{\lambda(1+2\gamma)} \qquad \ldots\ldots\ldots (27)$$

These results are obtained straightforwardly by following the steps outlined in comparison with the quartic potential. As we have seen ξ is redundant in the linear representation of the Starobinsky model, however we will retain the explicit dependence on ξ to ease the comparison between the two models.

We consider the action of a scalar field non-minimally coupled to gravity:

$$S_j = \int d^4x\sqrt{-g}\left[\frac{M_P^2+\xi\varphi^2}{2}R - \frac{1}{2}g^{\mu\vartheta}\delta_\mu\varphi\delta_\vartheta\varphi - \lambda\varphi^4\left(\frac{\varphi}{\Lambda}\right)^{4\gamma}\right]$$

We assume large field, inflationary regime $\varphi >> \frac{M_P}{\sqrt{\xi}}$

In this regime the non-minimal coupling term flattens the potential to an extent where slow-roll inflation is viable. The field redefinition approaches the solution

$$\chi = \sqrt{6}M_P \ln\frac{\sqrt{\xi}\varphi}{M_P^2}$$

for $\varphi >> \frac{M_P}{\sqrt{\xi}}$

And the Einstein frame potential takes the form

$$U(\chi) = \Omega^{-4}V(\varphi(x)) = \frac{M_P^4}{(M_P^2+\xi\varphi^2)^2}\lambda\varphi^4\left(\frac{\varphi}{\Lambda}\right)^{4\gamma}$$

$$\Rightarrow U(\chi) = \lambda\varphi^4\frac{M_P^4}{(M_P^2+\xi\varphi^2)^2}\left(\frac{\varphi}{\Lambda}\right)^{4\gamma}$$

$$\Rightarrow U(\chi) = \lambda\varphi^4\left(\frac{M_P^2+\xi\varphi^2}{M_P^2}\right)^{-2}\left(\frac{\varphi}{\Lambda}\right)^{4\gamma}$$

$$\Rightarrow U(\chi) = \lambda\varphi^4\left(\frac{M_P^2+\xi\varphi^2}{M_P^2}\right)^{-2}\left(\frac{\varphi}{\Lambda}\right)^{4\gamma}$$

$$\Rightarrow U(\chi) = \frac{\lambda\varphi^4}{\xi^2}\left(\frac{M_P^2+\xi\varphi^2}{\xi\varphi^2}\right)^{-2}\left(\frac{\varphi}{\Lambda}\right)^{4\gamma}$$

$$\Rightarrow U(\chi) = \frac{\lambda\varphi^4}{\xi^2}\left(1+\frac{M_P^2}{\xi\varphi^2}\right)^{-2}\left(\frac{\varphi}{\Lambda}\right)^{4\gamma}$$

$$\Rightarrow U(\chi) = \frac{\lambda\varphi^4}{\xi^2}\left\{1+\left(\frac{\sqrt{\xi}\varphi}{M_P}\right)^{-2}\right\}^{-2}\left(\frac{M_P}{\sqrt{\xi}\Lambda}\right)^{4\gamma}\left(\frac{\sqrt{\xi}\varphi}{M_P}\right)^{4\gamma}$$

$$\Rightarrow U(\chi) = \frac{\lambda\varphi^4}{\xi^2}\left\{1+\exp\left[\ln\left(\frac{\sqrt{\xi}\varphi}{M_P}\right)^{-2}\right]\right\}^{-2}\left(\frac{M_P}{\sqrt{\xi}\Lambda}\right)^{4\gamma}\exp\left[\ln\left(\frac{\sqrt{\xi}\varphi}{M_P}\right)^{4\gamma}\right]$$

$$\Rightarrow U(\chi) = \frac{\lambda\varphi^4}{\xi^2}\left\{1+\exp\left[\frac{-2}{\sqrt{6}M_P}\sqrt{6}M_P\ln\left(\frac{\sqrt{\xi}\varphi}{M_P}\right)\right]\right\}^{-2}\left(\frac{M_P}{\sqrt{\xi}\Lambda}\right)^{4\gamma}$$
$$\exp\left[\frac{4\gamma}{\sqrt{6}M_P}\sqrt{6}M_P\ln\left(\frac{\sqrt{\xi}\varphi}{M_P}\right)\right]$$

$$\Rightarrow U(\chi) = \frac{\lambda\varphi^4}{\xi^2}\left\{1+\exp\left[\frac{-2\chi}{\sqrt{6}M_P}\right]\right\}^{-2}\left(\frac{M_P}{\sqrt{\xi}\Lambda}\right)^{4\gamma}\exp\left[\frac{4\gamma\chi}{\sqrt{6}M_P}\right]$$

$$\therefore U(\chi) = \frac{\lambda M_P^2}{\xi^2}(1-\exp[\frac{-2\chi}{\sqrt{6}M_P}])^2.(\frac{M_P}{\sqrt{\xi}\Lambda})^{4\gamma}\exp[\frac{4\gamma\chi}{\sqrt{6}M_P}]$$

Φ4 –inflation correction from γ

The underbraced 'φ^4-Inflation'-term refers to the potential one would obtain by setting γ= 0, that is, non-minimally coupled φ^4-Inflation. As we have seen large field asymptotic flatness of this term makes non minimally coupled' φ^4 Inflation' viable. However, quantum corrections which we parameterize by may spoil this feature of the potential..

Conclusion

We considered the Starobinsky model of inflation and described how it is connected to matter scalar field models with non-minimal coupling, and at what level they dier. We considered quantum-induced marginal deformations of the Starobinsky action, and found that such deformations significantly shift the predicted tensor-to-scalar ratio towards higher values. At last we discussed sources for these corrections and argued that if inflation is driven by an f (R)-theory of gravity

$$U(\chi) \cong \frac{\lambda M_P^4}{4\xi^2}\left(1-\exp\left[\frac{-2\chi}{\sqrt{6}M_P}\right]\right)^2 \quad \ldots\ldots\ldots \text{(i)}$$

$$U(\chi) = \frac{\lambda M_P^2}{\xi^2}(1-\exp[\frac{-2\chi}{\sqrt{6}M_P}])^2.(\frac{M_P}{\sqrt{\xi}\Lambda})^{4\gamma}\exp[\frac{4\gamma\chi}{\sqrt{6}M_P}] \quad \ldots\ldots\ldots \text{(ii)}$$

Φ4 –inflation correction from γ

From the above we see that, when we consider the Starobinsky action principle in terms of a field y then we get the Einstein frame Potential as the form of equation (i) where there is only one exponential part of Positive Square.

But when we consider a Starobinsky action with a dimensionless parameter a then we get the equation (ii) where there are two exponential part-one is Φ4 –inflation part & other is a correction part from γ. Here the parameter γ is introduce here by using,

$$\chi = \sqrt{6}M_P\ln\frac{\sqrt{\xi}\varphi}{M_P^2} \quad \& \quad \varphi >> \frac{M_P}{\sqrt{\xi}}$$

Which is used in equation (ii). But in equation it appears as,

$$\chi = \sqrt{\frac{3}{2}}M_P\ln(1+\frac{R}{3M^2})$$

ie. If we let the field defined with respect to scale ratio (R) then we get the Staroibinsky Inflation Model & If we represent χ with respect to scalar field (φ) then we get the Starobinsky modified equation with respect to two exponent parts with inverse square.

This conclude that the Strobinsky & the dis-similarity between them & we finally established the equation as the Starobinsky modified equation.

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